Exam questions on the subject “Mathematical Analysis -II”

Module 1. Indefinite integral

1. Indefinite integral:
a) Formulate the definition of the antiderivative function f(x) on the interval (a,b);
b) Formulate the definition of an indefinite integral on the interval (a,b);
c) Integrals, expressible and not expressible in elementary functions.

2. Indefinite integral:
a) Basic properties of the indefinite integral;
b) Table of the simplest indefinite integrals (Integrals from power, exponential, trigonometric, rational functions).

3. Basic methods of integration:
a) Integration by reduction to tabular integrals using various transformations;
c) Substitution method (integration by change of variable).

4. Method of integration by parts:
a) Derive the formula $\int udv = uv - \int vdu$;

b) Types of integrals that are convenient to calculate by the method of integration by parts.

5. Using integration by parts, derive the recurrence formula for the integral $\int dx/(x^2 + a^2)^n$, $(n \in N)$

6. Integration of rational fractions:

a) Integrating a rational fraction $1/(ax + b)^n$

b) Integrating rational fractions $(Mx + N)/(ax^2 + bx + c)$

7. Integration of rational functions of the form P(x)/Q(x), where *P(x)* and *Q(x)* are entire algebraic polynomials of x.

8. M.V. Ostrogradsky’s method for integrating rational functions of the form P(x)/Q(x), where polynomial *Q(x)*, has multiple roots, including complex ones.
9. Integration of irrational expressions

- Integrating an expression of the form $\sqrt[n]{(ax+b)^m}$
- Integrating an expression of the form $\sqrt[n]{((ax+b)/(cx+d))^m}$

10. Integrating a differential binomial $x^m (a+bx^n)^p$; where $m, n, p \in Q$;

11. Calculation of integrals of the form: $\int R(x, \sqrt{a^2 - x^2})\, dx$, $\int R(x, \sqrt{a^2 + x^2})dx$, $\int R(x, \sqrt{x^2 - a^2})dx$

12. Universal trigonometric substitution in the integral of the form $\int R(\sin(x), \cos(x))\, dx$.

13. Integral of the form $\int R(\sin(x), \cos(x))\, dx$:

a) if the function R is odd with respect to $\cos x$;

b) if the function R is odd with respect to $\sin x$;

c) if the function R is even with respect to $\sin x$ and $\cos x$;

Module 2. Definite integral.

14. Definite integral

a) Formulate the definition of the integral sum for a given function $f(x)$ on the segment $[a,b]$;

b) Formulate the definition of the definite integral of the function $f(x)$ over the segment $[a,b]$.

15. Definite integral

a) Darboux sums and their properties for the function $f(x)$ on the segment $[a,b]$;

b) Conditions for the existence of a definite integral.

15. Definite integral

a) Darboux sums and their properties for the function $f(x)$ on the segment $[a,b]$;

b) Conditions for the existence of a definite integral.

16. Basic properties of a definite integral. Mean value theorem.

17. Integral with a variable upper limit. Newton–Leibniz formula.

18. Change of variable in a definite integral.

19. Formula for integration by parts in a definite integral

20. Geometric applications of the definite integral: Calculation of the area of a curved trapezoid.

21. Geometric applications of the definite integral: Calculation of the arc length of a curve.

22. Geometric applications of the definite integral: Calculation of the volume of a body of rotation.
23. The simplest quadrature formulas: Formula of rectangles.

24. The simplest quadrature formulas: Trapezoid formula.

25. The simplest quadrature formulas: Parabola formula (Simpson).

26. Improper integrals: Integral with an infinite integration interval (improper integral of the 1st kind).

27. Improper integrals: Integral of a discontinuous function
(improper integral of the 2nd kind).

Module 3. Series theory
28. Number series:
a) Concepts of number series, basic definitions
b) Geometric progression

29. Number series:
a) Properties of convergent series;
b) Cauchy criterion for the convergence of series;
c) A necessary sign of convergence of a series.

30. Number series with non-negative terms, signs of their convergence:
a) signs of comparison;
b) Generalized harmonic series (Dirichlet series).
31. Number series with non-negative terms, signs of their convergence:
radical Cauchy test, integral Cauchy test.

32. Number series with non-negative terms, signs of their convergence:

D'Alembert's sign, Raabe's sign.
33. Alternating number series, absolute and conditional convergence. A sufficient sign of convergence of an alternating series
34. Dirichlet and Abel tests for the convergence of alternating series.

35. Alternating number series, Leibniz’s test.

36. Basic properties of absolutely convergent number series.

37. Basic properties of conditionally convergent number series. Riemann's theorem.

38. The concept of an infinite product. Convergence.
39. Properties of infinite products.

40. Necessary and sufficient conditions for the convergence of infinite products.
41. Functional sequences and series:
a) Concepts of functional sequence, its convergence;
b) Concepts of functional series; its convergence at a point and on a set.

42. Uniform convergence of a functional series on a set. Cauchy criterion for uniform convergence of a series.

43. Functional sequences and series:
a) Weierstrass test for uniform convergence;
b) Dirichlet sign;
c) Abel's sign.

44. Statement of problems for the limit function and the sum of the functional series. Properties of the limit function and the sum of a functional series in the case of uniform convergence.

45. The concept of power series. Abel's theorem. Radius and interval of convergence of a power series.

46. Properties of power series.Разложение функции в степенные ряды. Ряд Тейлора.

47. Expansion of basic elementary functions in Maclaurin series